



Missing behavior data in longitudinal network studies: the impact of treatment methods on estimated effect parameters in stochastic actor oriented models

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Abstract

Research into missing network data is growing, with a focus on the impact of missing ties on network statistics or network model parameters. Longitudinal network studies using stochastic actor-oriented models (SAOMs) focus on the co-evolution of network structure and behavior/attributes to disentangle influence and selection mechanisms. Still little is known about the impact of missing behavior data on estimated effect parameters in SAOMs. This paper examines seven different methods that are currently available to deal with missing behavior data: complete cases, three single imputation procedures (imputing the mean, random hot deck, nearest neighbor hot deck), one multiple imputation procedure (based on predictive mean matching), and two methods available in the RSIENA software to estimate SAOMs (default method based on imputation and available cases, and a method based on dummy variables). In a simulation study based on four real-life data sets, the impact of these methods on estimated parameters of SAOMs was investigated. Missing behavior data were created under different conditions (proportions, mechanisms), and the missing data methods were used to estimate SAOMs on the incomplete data. The effect of the missing data methods was inspected using three criteria: model convergence, parameter bias, and parameter coverage. The results show that, in general, the default method available in the RSIENA software gives the best outcomes for all three criteria. The dummy-based method generally performed worse than the default method, as did the imputation procedures. The multiple imputation procedure sometimes outperformed the single imputations and the three single imputation methods often gave the same results. The effects of missing data mechanism and data set were small.

Keywords Missing data · Imputation · Longitudinal network · Stochastic actor-oriented models

1 Introduction

Social scientists often face the problem of missing data when analyzing empirically collected data. In the analysis of social networks, missing data constitute even a larger problem, because the complexity of collecting the network data and survey items are more likely to generate missing data (Burt 1987; Borgatti and Molina 2003). Moreover, due to the dependencies in the network, network analysis is especially sensitive to missing data, as the missingness not only limits the modeling of the local network of the actors involved, but

also limits the modeling of the local network structures of all neighboring actors (Robins et al. 2004).

In recent years, the effects of missing data in network studies are often studied, especially for cross-sectional data (e.g., Kossinets 2006; Žnidaršič et al. 2012; Smith and Moody 2013; Smith et al. 2017). The general conclusion that can be drawn from these studies is that missing data has a negative impact on describing and estimating the structural properties of the network, underestimating the strength of relationships, centrality measures, degree measures, or clustering coefficients (e.g., Kossinets 2006; Smith and Moody 2013; Smith et al. 2017). However, due to the unique property of networks that information on missing actors is (at least partially) available through the out-going ties of observed neighboring actors, measures based on indegrees are reasonably robust for small amounts of missing data (Costenbader and Valente 2003; Smith and Moody 2013).

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For longitudinal network data, where respondents are repeatedly observed, missing data are even more likely to occur. Some respondents will not be available at every observation moment, a situation known as wave non-response (see Huisman and Steglich 2008), or they will even drop out completely from the study after a certain time point. Huisman and Steglich (2008) studied the effect of missing longitudinal network data within the framework of stochastic actor-oriented models (SAOMs), a family of models often used to analyze the dynamics of network and behavior (Snijders et al. 2010b). They found that restricting the analysis to completely observed cases leads to model convergence problems and generally gives biased parameter estimates. Non-convergence due to missing data was also encountered by de la Haye et al. (2017) while analyzing the complete cases, which lead them to propose and study different analytic strategies for longitudinal networks with missing data.

Simple treatment procedures for missing network data were already suggested by Burt (1987) and Stork and Richards (1992). More recent, model-based procedures were proposed by Robins et al. (2004), Handcock and Gile (2010), Koskinen et al. (2010, 2013), all based on modeling observed and missing data within the framework of exponential random graph models (ERGMs). Imputation-based procedures were proposed and studied by Huisman (2009), Wang et al. (2016), Huisman and Krause (2017), and Krause et al. (2018b). All these procedures treat missing actors and ties in cross-sectional network data. For longitudinal network data, missing data procedures for analyses based on SAOMs were investigated by Huisman and Steglich (2008), Hipp et al. (2015), and de la Haye et al. (2017). Huisman and Steglich (2008) use simulations to study simple imputation schemes, one of which is the built-in (default) missing data treatment in SIENA, the software to estimate SAOMs (Ripley et al. 2017). This SIENA method was found, in general, to result in small biases in model parameters for small to medium missingness levels (up to 20% per wave). In the studies of Hipp et al. (2015) and de la Haye et al. (2017), analytical strategies are proposed that are based on inclusion of different subsets of actors depending on the availability of data in particular waves. Some of the strategies rely on the simple default imputations in SIENA, and one of the strategies proposed by Hipp et al. (2015) expand these simple imputations by including ERGM-based imputations for missing values in the first wave. This procedure creates the opportunity for multiple imputation (of the first wave), as suggested by Hipp et al. (2015). Krause et al. (2018a) present a full multiple imputation procedure based SAOMs.

Although research in missing data procedures for social networks is increasing in the past decades for both cross-sectional and longitudinal data, all methods are designed to treat missing ties in the network data and very few do address the problem of missing actor behaviors or behavior

data. Missing actor behavior could be regarded as “ordinary” missing data in any non-network data set, and thus treated as such by one of the ample general missing data methods for survey data available in statistical literature. However, treating missing behavior without considering their (often strong) relationship with the structural properties of the network, may bias effects of behavior and may lead to biased estimates of the structural properties. Koskinen et al. (2013) illustrate the effect of missing behavior data and present an ERGM-based procedure to analyze the incomplete data. Ouzienko and Obradovic (2014) propose an ERGM-based imputation procedure for the case of longitudinal network data (i.e., temporal ERGMs). In a small simulation study, using simulated and real-life data, they showed that, in general, their imputations result in more accuracy in predicting tie and behavior variables (comparing observed and imputed scores) than simpler methods.

In this paper, we investigate the performance of several treatment methods to handle missing behavior data in longitudinal networks. More specifically, we analyze the impact of different treatment methods on estimated effect parameters in SAOMs that are used to model the dynamics of network and behavior (Snijders et al. 2010b). We restrict the missingness to the behavior variable, that is, the networks are completely observed. This means that any effect found can only be attributed to the missing behavior data and is not confounded by missing ties or actors. The network and behavior data are simulated under known coevolution models (the base models in our study) and we examine missing data strategies that are available for SAOMs (complete case analysis, dummy variable adjustment in SAOMs, SIENA method) and some simple, ad hoc treatments (i.e., simple single imputations, based on means and hot deck, and somewhat more sophisticated multiple imputations using observed network statistics as predictors). The simulated data are based on four empirical, observed data sets, in the tradition of Smith and Moody (2013), Smith et al. (2017), and others. In that respect, the current paper can be regarded as a continuation of the study of Huisman and Steglich (2008), focusing on missing behavior data.

The paper is organized as follows. Section 2 briefly describes the stochastic actor-oriented models for the dynamics of networks and behavior (Snijders et al. 2010b) that are used to simulate the data sets and analyze the treated missing data to examine the effects of missing data treatments. Section 3 addresses the problem of missing data in networks and especially in behavior data and introduces the available missing data treatments of which the performance is studied. The design of the simulation study is described in Sect. 4, and the results are presented in Sect. 5. Finally, in Sect. 6 the results are discussed and some general recommendations are given.

2 Stochastic actor-oriented models

A common model to analyze the dynamics of networks and behavior is the family of stochastic actor-oriented models (SAOMs); (Snijders 2005, 2017; Snijders et al. 2010a), of which the estimation is implemented in the SIENA software (SIENA package, Ripley et al. 2017). In this paper, we consider directed networks, where the tie variable x_{ij} is binary with values 1 (indicating a tie going from actor i to actor j) or 0 (absence of a tie between actors i and j). For example, the tie variable is friendship, where $x_{ij} = 1$ means that actor i nominates actor j as a friend. Self-nominations are not allowed, that is, $x_{ii} = 0$. The behavior variable is assumed to be an ordinal discrete variable representing levels of some behavior (e.g., smoking). In the SAOM approach, the network dynamics part in the coevolution process constitutes the social selection process, and the behavior dynamics part constitutes the social influence process.

Stochastic actor-oriented models (SAOMs) model the coevolution of network and behavior. A SAOM is based on panel data, and assumes that the observed data are snapshots of an underlying and unobserved process of continuous change in network and behavior between the observation moments. This change process is modelled as a continuous-time Markov chain of small sequential mini steps, where the first observation is taken as starting point. Each mini step gives a randomly selected actor the opportunity to change either a tie or the value of the behavior variable. For the tie variable, a change means adding a tie to another actor or dropping an existing tie, or no change. For the behavior variable, a change means increasing or decreasing the value with one unit, or no change. See Adams and Schaefer (2018) for a visualization of the model mini steps.

The change processes consist of two steps. First, a stochastic rate function determines when an actor gets the opportunity for a new change (mini step). Secondly, the probabilities of the changes for both tie and behavior variables are determined by objective functions that are modeled as linear combinations of effects that represent the current network structure and behavior. These effects are functions of the network of the focal actor, as well as the behavior of that actor and the behavior of his network partners. Because the changes in the network are also dependent on the state of the actors' behavior, and vice versa, a mutual dependence between the network dynamics and the behavior dynamics is established. Examples of effects and the corresponding parameters are given in Sect. 4; for a more elaborate discussion of the objective functions and examples and illustration of effects, see Snijders et al. (2010a) and Snijders (2017).

Because the mini steps between observed measurements are unobserved, a SAOM is used to simulate the Markov

chains of mini steps. The simulation starts with the first observation of network and behavior (W1), and, using an initial set of model parameters, simulates changes until the second observation (W2). Based on a comparison of the simulated data at W2 and the observed data at W2, the model parameters are updated. With these updated parameters the simulations are repeated. This iterative process is repeated until the model has reached convergence. After convergence, the final parameter estimates are used to generate additional series of simulated mini steps to estimate standard errors of the parameter estimates (for details see Snijders 2001, 2017).

3 Non-response in longitudinal network studies

3.1 Missing behavior data

In this paper, we focus on missing data due to non-response. Other types of missing network data are described by Kossinets (2006) and Žnidaršič et al. (2012), for example missingness caused by boundary specification problems. We consider two observation moments, where the networks are completely observed and one behavior variable that is missing for some actors at both observation moments. The non-response pattern is important, because it determines the amount of data available to estimate the SAOMs.

Another important aspect of the non-response is the relationship of the missingness to the data. According to the typology of Rubin (1976; see also Schafer and Graham 2002), three different mechanisms can be distinguished, depending on the relation between being missing on a behavior variable and the scores on (the behavior or tie) variables. If the missingness is unrelated to the value of the behavior variable itself, the data are called *missing at random* (MAR). In this situation, the non-response can be related to the observed tie variables, or function thereof, but not to the behavior itself. If the missingness is even unrelated to the observed tie variables (or, in general, to any other variable in the data), the data are called *missing completely at random* (MCAR). If the missingness is related to the unknown value of the behavior itself, the data are *missing not at random* (MNAR). In this latter situation, parameters related to the behavior may be severely biased due to the systematic difference between responding actors and non-respondents.

3.2 Treatments for missing behavior data

In recent years, missing data treatment procedures have received ample attention, for both cross-sectional and longitudinal network data. In general, missing data treatments can roughly be categorized in three classes of treatments (e.g., Schafer and Graham 2002)¹: (1) deletion methods (also known as available case methods), (2) model-based methods, and (3) imputation. Model-based methods for missing cross-sectional network data were proposed by Robins et al. (2004), Handcock and Giles (2010), Koskinen et al. (2010, 2013), all within the family of exponential random graph models. Imputation methods for cross-sectional network data were proposed and examined by Huisman (2009), Wang et al. (2016), Huisman and Krause (2017), and Krause et al. (2018b), and for missing longitudinal network data by Huisman and Steglich (2008), Ouzienko and Obradovic (2014), Hipp et al. (2015), and Krause et al. (2018a). A combination of available case strategies and imputation within SAOMs (i.e., the default method implemented in the SIENA software) was examined by Hipp et al. (2015), de la Haye et al. (2017), and Krause et al. (2018a).

The problem of missing actor behavior data has received far less attention in network analysis. One possible strategy to handle the non-response is treating the behavior or behavior variables as “ordinary” survey data and using general missing data methods. The advantage of this strategy is that general missing data treatments have been investigated extensively and there are well-known and sophisticated methods, for example, multiple imputation using stochastic regression imputation with actor attributes or other behavior variables as predictors (as illustrated for actor behavior data by Huitsing et al. 2014). A major disadvantage is that the network structure is not taken into account and the associations between behavior and ties are ignored. Unless network and behavior are completely independent, this can lead to biased estimates of these relationships as well as biases in the estimates of network properties. To prevent the results from becoming biased, either network properties should be incorporated in general missing data procedures, or missing data treatments should be based on network models.

For missing behavior data, an ERGM-based estimation method was proposed by Koskinen et al. (2013). In this method, ERGMs are estimated on partially observed data (both network and behavior data) using Bayesian procedures that take into account the relations between network and behavior. Imputation methods for behavior data are scarcely investigated. Ouzienko and Obradovic (2014) present an

ERGM-based imputation model for imputing both missing tie variables and missing actor behaviors for longitudinal network data. For missing behavior variable in SAOMs, Ripley et al. (2017) propose a simple imputation scheme in which either the previous observation, the next observation, or the mode of the variable is imputed, in order of availability. These imputed values are then used to simulate the mini steps that constitute the behavior (and network) dynamics, but not for the calculation of the target statistics to estimate the model parameters, preventing a direct effect of the imputed values on model estimation.

In this paper, we consider procedures that are currently available to handle missing behavior data within the SAOM framework. This means that we investigate the possibilities in the SIENA software and compare these with either simple (complete case analysis or single imputation) methods, or with more elaborate multiple imputation methods in which the missing behavior variable is regarded as “ordinary” survey data in non-network analyses. Specific details about the use of the methods in the simulation study are given in Sect. 4.

3.2.1 Complete case analysis

Complete case analysis is based on the smaller network of complete cases. This means that all actors with missing behavior data are removed from the analysis, including the ties to or from them. The reduction in the data can be considerable and the results of the method will be highly sensitive to the proportion missing data. This may result in biased estimates of network characteristics even if data are MCAR. Moreover, model estimation is difficult if the remaining complete data set is small and may lead to convergence problems.

3.2.2 Single imputation

To avoid the loss of data due to complete case analysis, the missing data can be replaced by suitable values to create a completed data set. A simple procedure is to replace the missing values by the mean of the observed data. Although this method is simple and, therefore, attractive, it will lead to biased estimates even when data are MCAR, as it seriously underestimates variances and covariances (e.g., Schafer and Graham 2002). To preserve variation in the data, imputations can be generated by drawing from the distribution of the (missing) data. One way to generate such distributions, is using observed donor cases and replacing the missing values by the observed values of the donors. These methods are known as hot deck imputations. Although hot deck partially solves the problem of underestimating variances, it still gives biased results for relations between variables (effects).

¹ Often a fourth class of treatments is distinguished, i.e., (re)weighting procedures, which are not considered for missing network data.

3.2.3 Multiple imputation

A drawback of single imputations is that they do not take into account the extra variability due to missing data and imputation. This leads to underestimation of standard errors and, therefore, biased inferences. By imputing multiple times, the increased variability is accounted for and valid inferences are obtained (Rubin 1987; Schafer and Graham 2002; Van Buuren 2012). With multiple imputation, m ($m > 1$) completed data sets are created using stochastic single imputation methods. This leads to m completed data sets, which will be different from each other due to the stochastic nature of the imputations, the extent of which reflects the uncertainty due to missing data and imputation.

After imputation, the m completed data sets are analyzed separately (i.e., the parameters of the specified model are estimated for each of the data sets) and the results are combined using Rubin's rules (Rubin 1987). For parameter estimates, this simply means averaging the m parameter estimates for each imputed data set: $\theta = \frac{1}{m} \sum_{i=1}^m \hat{\theta}_i$, where $\hat{\theta}_i$ is the estimated parameter for the i th imputed data set. For the variances of the estimates (i.e., standard errors), the average within-imputation variance is combined with the between-imputation variance to reflect increased variability due to non-response and imputation: $T = \bar{U} + \left(1 + \frac{1}{m}\right)B$. Here $\bar{U} = \frac{1}{m} \sum_{i=1}^m U_i$ equals the average variance within each imputed data set, with U_i the variance in each imputed data set, and $B = \frac{1}{m-1} \sum_{i=1}^m (\hat{\theta}_i - \theta)^2$ equals the variance between the m estimated parameters. The factor $\frac{1}{m}$ in the equation of the total variance T reflects the finite number of imputations. Standard errors for parameters are obtained by taking the square root of the variance T .

3.2.4 SIENA procedures

The last two methods investigated in the simulation study, are procedures within the framework of SAOMs that are available in the SIENA software. The first procedure is the model-based hybrid imputation method for ties (Huisman and Steglich 2008) extended to behavior variables, the default procedure for missing data treatment in the SIENA software. The method is called hybrid, because in estimating the SAOM parameters, it uses imputed values during the simulation of the Markov chains of mini steps, but during the calculation (updating) of the estimates, the imputations are not used. This means that during the simulation of the Markov chains of mini steps between two consecutive waves, all actors (observed and missing) are allowed to make changes. At the end of the simulation runs when the simulated and observed data of the second time point are compared, the parameter updates are based on the observed data

only, and imputations are not taken into account. As a result, the imputations only have indirect effects on the estimates through the Markov process in the simulation phase of the procedure. In the default procedure, imputation consists of replacing missing values with previous observations from an earlier wave, if available, otherwise the mode of the variable (for the corresponding wave) is imputed.

The second procedure investigated in the simulation study is handling missing behavior data using dummy effects in the SAOM. In this procedure, a dummy variable is created that indicates whether an actor is missing (value 1) or observed (value 0). The dummy is included in the SAOM by specifying a dummy effect in the objective function of the behavior part of model, where the value of the parameter is fixed at a large negative value. In this way, a large (artificial) negative effect on the objective function of the missing actor is created if this actor would choose to make a change in the behavior variable (i.e., take a mini step in the Markov process modeling the behavior dynamics). As a result the actor will decide not to change his behavior. The missing actor will thus not influence the behavior dynamics in the model.

This dummy variable procedure differs from the traditional dummy variable adjustment of missing values in regression models. In the traditional setting, a dummy variable is created indicating missing values on the predictor and a new predictor variable X^* is constructed with values equal to X for the observed cases, and c (any constant) for the missing cases. The estimated parameter of the dummy variable represents the influence of missing predictors on the outcome variable. The estimated parameter for the new predictor X^* represents the estimated effect of X for the observed cases. This procedure redefines the parameters estimates and generally produces biased estimates of the coefficients (Allison 2001; Schafer and Graham 2002).

4 Simulation study

To investigate the impact of various missing data treatments on the parameter estimates of the SAOMs, a simulation study was performed. In this study, a modified version of the general pattern of the simulation study by Huisman and Steglich (2008) was followed:

1. Select a data set consisting of both network and behavioral data. In line with Smith and Moody (2013) and Smith et al. (2017), data sets representing a variety of commonly studied networks were selected, limited to small networks (smaller than 65 nodes), which are typically found in empirical research using SAOMs.
2. For each data set, estimate a SAOM, the so-called base model, on the first two observed waves. This base model

Table 1 Sample network descriptive statistics for all three data sets (S50, G58, L63, H57): Network size, density, average degree, Moran's I, the Jaccard index, and relative frequencies of the categories of the behavioral variable

Network	s50		G58		L63		H57	
	Wave 1	Wave 2						
Network size (n)	50		58		63		57	
Density	0.05	0.05	0.05	0.04	0.38	0.39	0.10	0.10
Average degree	2.26	2.32	2.69	2.33	23.4	24.1	5.32	5.70
Moran's I	0.43	0.40	0.33	0.05	0.12	-0.04	0.05	0.03
Jaccard Index	0.33		0.30		0.67		0.39	
Behavior relative frequencies								
1	0.10	0.06	0.05	0.03	0.27	0.19	0.14	0.14
2	0.32	0.32	0.48	0.33	0.19	0.24	0.14	0.07
3	0.24	0.24	0.29	0.40	0.02	0.27	0.37	0.38
4	0.28	0.22	0.14	0.21	0.00	0.11	0.21	0.24
5	0.06	0.16	0.03	0.03	0.00	0.13	0.14	0.15
6	-	-	-	-	0.52	0.06	0.00	0.00

represents the 'true' model and is based on the complete data set, before generating missing actors.

- Using the selected data set and the base model, generate complete (i.e., non-missing) sets of longitudinal data consisting of two waves.
- Generate missing data in both waves by deleting the behavioral data of a fraction of the actors. Note that the network data are not deleted.
- Use the procedures outlined in Sect. 3.2 to handle the missing data and re-estimate the SAOM.
- Investigate the effect of missing data handling on the estimation procedure and the estimated parameters of the SAOM by comparing the parameters of the re-estimated models after treating for the missing (deleted) behavior, with the parameters of the base model. This comparison is based on the following three criteria: convergence of the estimation procedure, parameter bias, and parameter coverage.

Details and specifications of various steps in the simulation process are given in the following subsections.

4.1 Selection of data sets and generation of longitudinal data

Four different data sets were selected to be used in the simulation study. Each data set consists of at least two waves of network and behavioral data. To limit computation time and convergence problems, from each data set, the first two waves of one network and one behavioral variable are used. The sampled networks are similar in size, ranging from 50 to 63 actors, and consist of friendship or advice relations. Data sets one and two are subsets of the friendship networks from the *Teenage Health and Lifestyle* study (Michell and Amos 1997; Pearson and West 2003). The first consists of 50 girls (labeled s50) with the behavioral dependent variable alcohol

consumption, which is coded on a five-point frequency scale ranging from 1 ('I don't drink') to 5 ('I drink more than once a week'). This data set was also used by Adams and Schaefer (2018) for a visualization of the mini steps in SAOMs and in the simulation studies of Huisman and Steglich (2008), Huisman (2009), and Krause et al. (2018a). The second data set consists of a subset of 58 boys from the same study (labeled G58), also with friendship defining network ties and alcohol consumption as dependent behavioral variable.

The third data set comes from a study among 63 managers of an international company (labeled L63; Zandberg et al. 2018). It consists of two waves of an advice network and the behavioral variable is information synthesizing, which involves gathering, evaluating, and distributing strategic information to the top management of an organization (Floyd and Wooldridge 1997). Synthesizing is coded on a six-point frequency scale ranging from 1 ('hardly synthesizing information') to 6 ('regularly synthesizing'). The fourth data set (labeled H57) comes from a study among 57 staff members of a housing corporation in the Netherlands (Whitmeyer and Wittek 2010). It consists of an advice network and dependent behavioral variable stress at work. The behavioral variable is coded on a five-point frequency scale ranging from 1 ('not or hardly ever stressed at work') to 5 ('very often or always stressed at work').

Some data characteristics of the first and second observation of the data are presented in Table 1, and a visual presentation of the first wave of each network is given in Fig. 1. In Table 1, density is calculated as the number of actual ties divided by potential number of ties, and degree is the number of ties of each actor. Moran's I is the spatial autocorrelation index that measures the association between behavior and network (i.e., the correlation of behavior between actors that are related to each other). The Jaccard index is the Jaccard distance between two successive networks (wave 1 and wave 2) and measures stability between the waves.

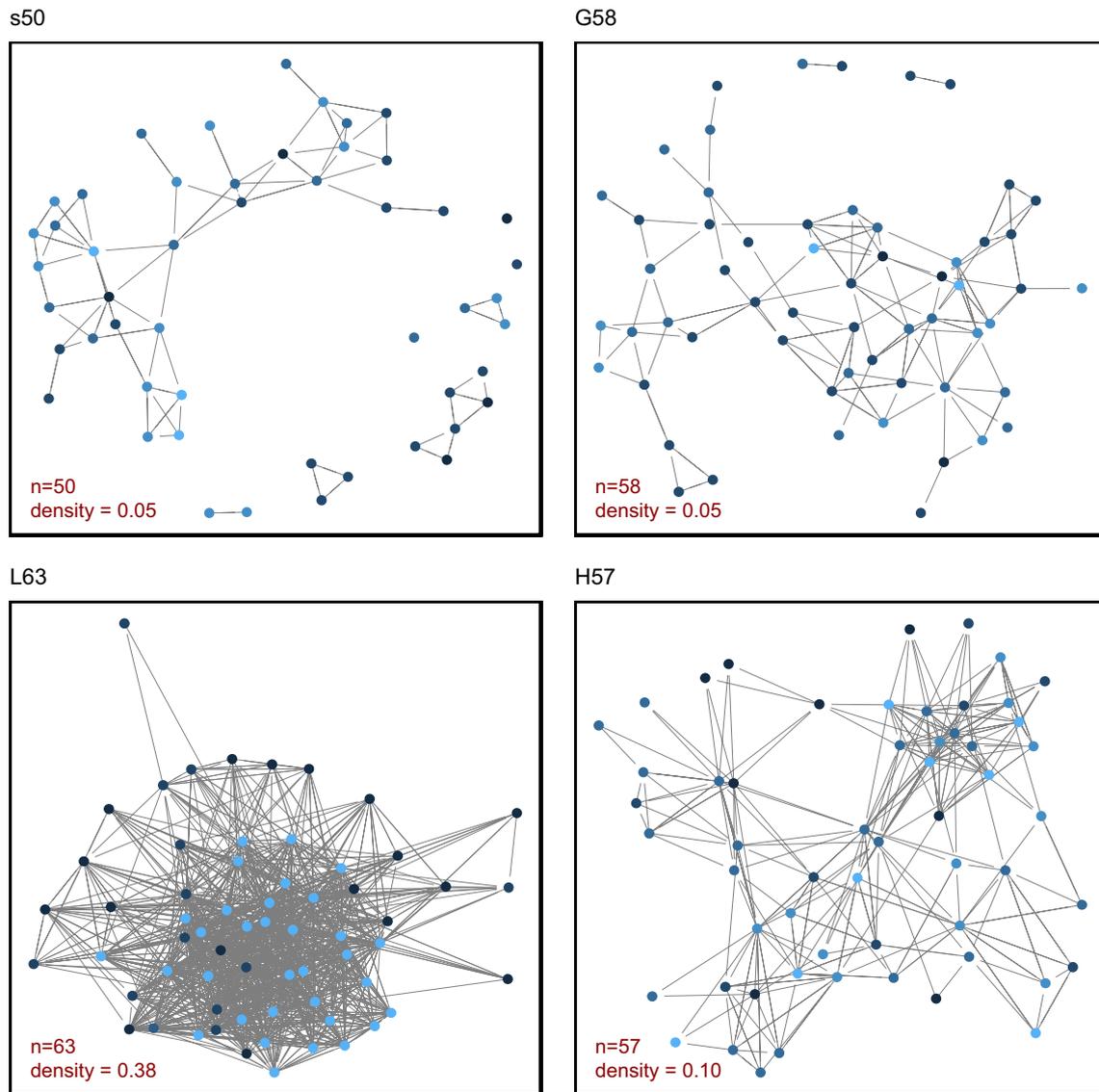


Fig. 1 Graphs (wave 1) of the networks used in the simulation study: Friendship networks s50 and G58 (top row) and advice networks L63 and H57 (bottom row)

The networks are comparable in terms of size, but differ in density, with the friendship networks s50 and G58 having the lowest densities. The advice networks are denser, with the H57 network showing some cliques and some actors in (very) central positions. The L63 network is rather dense showing high levels of interaction between the actors. In the s50 data, Moran's I, the network autocorrelation, equals 0.43 and 0.40 for wave 1 and wave 2, respectively, which means there is a strong association between network and behavior. In the G58 data, Moran's I decreases from 0.33 to 0.05, signifying a decrease in association between network and behavior. In the L63 data, Moran's I equals 0.12 and -0.04 , for wave 1 and 2, respectively, and 0.05 and 0.03 in the H57 data, which signifies a rather low association

between the network and behavior in both data sets (Veenstra et al. 2013). The Jaccard index measures the amount of change in the network between two waves, and should be large enough to provide enough information to estimate the parameters. A value of 0.3 is usually considered adequate (Ripley et al. 2017). The Jaccard index varies from 0.30 to 0.67, indicating there is enough change between the waves to enable estimation of a SAOM in all data sets.

The first two waves in each data set were used to generate simulated coevolution processes of networks and behavior. On each data set, a SAOM was estimated that is used as the base or 'true' coevolution model to generate the data in the study. These base models are presented in Table 2. As the data sets differ in type of network and

Table 2 Specification of base models to generate longitudinal data for networks and behavior for the four data sets: estimated parameters b (with standard errors) and convergence t statistics

Effect	s50		G58		L63		H57	
	b (SE)	t	b (SE)	t	b (SE)	t	b (SE)	t
1 Network rate	5.94 (0.99)	-0.01	5.78 (0.70)	-0.02	14.63 (0.90)	0.01	7.68 (0.64)	0.02
2 Density	-2.59 (0.17)	0.01	-2.49 (0.23)	0.02	-1.70 (0.08)	-0.01	-1.36 (0.07)	0.00
3 Reciprocity	2.06 (0.26)	0.02	2.27 (0.29)	0.00	1.17 (0.09)	-0.02	0.99 (0.14)	0.00
4 Transitive triplets	0.61 (0.12)	0.02			0.04 (0.00)	-0.02		
5 Behavior alter	-0.14 (0.11)	-0.05	0.58 (0.28)	0.03	-0.20 (0.05)	-0.02	0.05 (0.05)	0.03
6 Behavior ego			-0.15 (0.28)	0.01			-0.12 (0.06)	0.01
7 Behavior similarity	1.60 (0.75)	-0.03	2.10 (1.60)	0.01			-0.78 (0.32)	0.04
8 Behavior rate	1.02 (0.40)	0.03	1.76 (0.54)	-0.01	6.64 (1.59)	0.02	0.92 (0.27)	0.02
9 Behavior linear	-0.35 (1.23)	0.05	-0.38 (0.54)	0.04			-0.20 (1.01)	0.03
10 Behavior quadratic	-0.16 (0.36)	0.00	-0.21 (0.20)	0.01			0.61 (0.75)	0.00
11 Behavior total similarity			2.36 (1.85)	0.03			2.26 (2.95)	0.00
12 Behavior indegree	-0.69 (1.46)	0.03	0.01 (0.43)	0.03			-0.05 (0.20)	0.03
13 Behavior outdegree	1.10 (2.16)	0.03	0.29 (0.54)	0.04	-0.05 (0.02)	0.01	0.17 (0.31)	0.02
Overall convergence t statistic		0.10		0.09		0.06		0.06

The t statistic is the average deviation between simulated values of the statistics and their target values in the final phase of the estimation phase, where the standard errors of the statistics are estimated

behavior, we tried to keep the base models as similar as possible using the following set of standard effects:

The first three effects specify the dynamics of the network.

- Density (outdegree), the basic tendency of actors to have ties.
- Reciprocity, the tendency of relations to be returned. If actor A asks actor B for advice, this increases the probability of B asking A for advice.
- Transitive triplets, the tendency to form transitive triplets. If actor B is a friend of actor A, and actor C is a friend of actor B, the probability of A becoming a friend of C will increase (friends become friends with their friends' friends).

The following three effects model the influence of behavior on network structure.

- Behavior alter describes the effect of behavior on the actor's popularity to attract other actors; a positive parameter indicates a tendency that actors with high levels of behavior will receive more incoming tie requests. For example, spending lots of money might be a reason for getting befriended.
- Behavior ego describes the influence of an actor's level of behavior on extending ties to others. For example, being successful leads to more easily approaching others.

- Behavior similarity describes the effect of forming a tie with actors with similar levels of behavior, like non-smokers befriending non-smokers.

The following three effects model the influence of network structure on behavior.

- Behavior total similarity describes the actors' preference to be similar to their alters.
- Behavior indegree describes the tendency that popular actors (with more incoming ties) have higher values for behavior.
- Behavior outdegree describes the tendency that more active actors (with more outgoing ties) have higher values for behavior.

In a first attempt, a model containing all the described effects was estimated on each data set. To obtain acceptable convergence results for all data sets, in a second round some effects were removed from the model of some data sets. This resulted in simple base models that have slightly different specifications for all data sets, good convergence qualities, do not take too much computing time, and are able to generate empirically informed simulations. A drawback is that some parameters are not significant. All four base models satisfy the common convergence criteria (Ripley et al. 2017), with convergence statistics for individual parameters smaller than 0.10 and t statistics for overall convergence smaller than 0.25. It should be noted

that the satisfying convergence of the base models is also due to the relatively simple model specifications.

The four estimated base models are used to simulate the coevolution processes. The first observation of each network and corresponding behavioral variable are taken as initial state of the process and using the estimated model parameters, the coevolution process was simulated 500 times. This resulted in 500 simulated networks and 500 simulated behavioral variables at the second time point. These simulated data (network and behavior) are taken as wave two data in the simulation study (after generating missing data).

4.2 Generating missing data

As the study is restricted to missing behavior (endogenous) variables, missing data were created by selecting actors according to some stochastic procedure and deleting the values of the behavioral variable of these actors. Four proportions of non-response were generated: 0.1, 0.2, 0.4, and 0.6. For each proportion, actors were sampled using one of the selection mechanisms described below, and missing values were created in the behavior variable at both time points. That is, an actor with non-response on behavior, has missing values on both time points. In line with Huisman and Steglich (2008) and using the typology defined by Rubin (1976), we used three different mechanisms to select the actors with missing behavioral values.

1. Missing completely at random (MCAR): Completely random selection, where missingness is not related to characteristics of the network or the actors.
2. Missing at random (MAR): Probability of selection is proportional to $1/(\text{outdegree} + 1)^2$, where missingness is related to an observed network characteristic.
3. Missing not at random (MNAR): Probability of selection is proportional to $1/(\text{behavior} + 1)^2$, where missingness is related to the behavioral variable itself.

In the second and third mechanism, the selection is such that actors with lower scores on the characteristic (outdegree and behavior, resp.) have a larger probability to have missing data. While these may not be the only realistic mechanisms to select actors with missing behavior data, they are very suitable to illustrate the impact of the different types of mechanisms.

The generation of the observed and missing data resulted in $4 \text{ (data sets)} \times 500 \text{ (replications)} = 2000$ complete data sets (two waves of network and behavior two waves), and $4 \text{ (data sets)} \times 500 \text{ (replications)} \times 4 \text{ (proportion missing)} \times 3 \text{ (missingness mechanism)} = 24,000$ incomplete data sets.

4.3 Treatment of missing data and re-estimation of SAOM

For each network, the data were analyzed using the SAOM that was used to generate the data (see Table 2). Next, the same SAOM was fitted to the incomplete data, where the missing behavior data were treated using one of the missing data procedures described in Sect. 3.2:

1. Complete cases (CC).
2. Single imputation by imputing the mean of the observed data (AV).
3. Single imputation by imputing the value of a randomly selected donor case (RAN). For each missing actor, a donor case is selected at random from the set of observed actors and the value of the behavior variable of the donor actor is used to replace the missing data.
4. Single imputation by imputing the value of a selected donor case (hot deck imputation; HD). As in the previous procedure, the missing behavior is imputed by the behavior value of a donor actor. The donor actor, however, now is an actor resembling the actor whose behavior is missing. The selection of a matching donor is based on the absolute difference in outdegree between the actor with missing behavior data and the donor actor (the smaller the difference, the higher the probability of the donor to be selected).
5. Multiple imputation based on predictive mean matching (MI). Predictive mean matching (Little 1988) is a hot deck procedure in which donor actors are selected of which the observed values are imputed. The selection of the donors is based on matching predictions from regression models. In the case of missing behavior data, a set of three observed donor actors is found of whom the predicted behavior scores are close to the predicted value of the missing actor, and from this set one donor is randomly drawn (van Buuren 2012). The regression models on which the predictive mean matching is based, consist of additional behavioral or attribute variables and network effects, depending on the data set that is imputed (one of the four described in Sect. 4.1). For all four networks, the network statistics that were used in the imputation model are indegree, outdegree, number of reciprocal ties, number of transitive ties, number of three-cycles, and number of two-paths. The additional behavior variables that were used in the imputation model are use of tobacco, use of cannabis, sports participation (s50 data set), use of tobacco, use of cannabis, amount of pocket money, having a romantic relation, distance to school (G58 data set), proactive behavior, discretionary space, organizational support, organizational commitment, and organizational connectedness (L63 data set). For the H57 data set no additional behav-

ioral variables were used as predictors in the imputation model, as none were available. The multiple imputation for each incomplete data set was performed using the R software package *mice* (van Buuren and Groothuis-Oudshoorn 2011), based on $m=5$ imputations. These imputations were simulated until five convergent runs were obtained. When less than five convergent runs were obtained after twenty-five attempts, simulation was stopped. In that case, if we had achieved four convergent runs, multiple imputation was based on these four imputations, and in case we had obtained less than four convergent single imputations, this particular simulated data set was discarded for further analysis.

6. The default SIENA method (SIENA) based on imputation and restricted parameter estimation (as described for ties by Huisman and Steglich 2008).
7. The dummy variable procedure for SAOMs (DUM). The parameter for the effect of the dummy variable (indicating missing actors) was fixed at the value -40 , which proved large enough (in absolute value) to prevent missing actors from making a change in the dynamic processes.

4.4 Analysis of the simulation outcomes

Three measures of performance were used to investigate the effect of the missing data procedures for behavioral variables on modeling the longitudinal data: convergence (number of converged estimation runs), relative bias (compared to true score, i.e., the parameter of the data-generating SAOM), and coverage (percentage of intervals containing the true parameter value). A convergent simulation is a prerequisite for a reliable parameter estimation in SIENA. Convergence indicates that the statistics of the simulated networks in the estimation procedure, are close to their target values. In our study, we considered a simulation run converging if after one simulation run the t statistic for overall model convergence is smaller than 0.25 (Ripley et al. 2017). While one usually reruns a simulation when convergence is not satisfactorily in a first attempt, we did not rerun simulations as we considered the number of convergent runs in a first attempt a good indication of the effect of a missing data treatment on convergence characteristics. For multiple imputation, the approach is slightly different, because MI is based on five (or four) convergent sub runs. Therefore, obtaining a MI-result for a specific run automatically implies that it is based on convergent sub runs. The difference with the other methods is that we had more opportunities in obtaining convergent runs. If a run was not converging, it was discarded for further analysis.

To evaluate the robustness of the missing data handling methods, the relative bias of the estimated parameters was calculated: $\text{bias} = (\text{treated} - \text{true})/\text{true}$, where *treated* is the

estimated parameter for the treated missing data, and *true* is the estimated parameter in the original base model. Because for some combinations of data set, parameter, mechanism, and method, the number of convergent simulation runs was very low, only those combinations with at least 100 converging runs were considered.

To evaluate the distribution of the estimated parameters, the proportion of population parameters (base model) within two-standard-errors distance from the estimated parameter was calculated. In case of a normal distribution of estimated parameters, this distance would be smaller than two standard errors in approximately 95% of the cases. Although the distribution of estimated parameters in SAOMs is unknown, we will use this procedure to approximate parameter coverage. Similar to the relative biases, this is based on combinations with at least 100 convergent runs only.

5 Results

5.1 Convergence

The results are presented in Fig. 2, which shows the number of convergent simulation runs for each combination of data, method, missingness mechanism, and level of missingness. Each combination has been simulated 500 times, and the graphs show how many of these 500 runs were convergent. For example, in the G58 data set, the number of convergent simulations is approximately 380 when there is no missing data, but below 100 for 60% missing data and complete case analysis (CC).

First, we observe that the missingness mechanism hardly matters in most cases. In the G58 data set, mean imputation (AV) shows the worst performance for the MAR mechanism, while multiple imputation (MI) performs worst for MCAR. In the L63 data set, MAR generally shows lowest convergence rates (especially for the imputations based on donor cases: RAN, HD, and MI) for high levels of missingness. As a general conclusion, we can say that the missingness mechanism seems to have only a small impact on convergence.

When we look at the convergence characteristics of the different methods, it can be observed from Fig. 2 that for all data sets CC leads to low numbers of convergent simulations for higher levels of missingness. This is no surprise as the number of network relations declines exponentially and the remaining number of actors and ties quickly becomes too small. The implication is that CC is not a useful method to deal with missing data. The single imputation methods AV, RAN, and HD replace the missing data by a value that is based upon the observed data. It is, therefore, no surprise that their convergence results show similar patterns. Multiple imputation shows patterns that are often similar to mean imputation (AV) and is frequently hardly better than

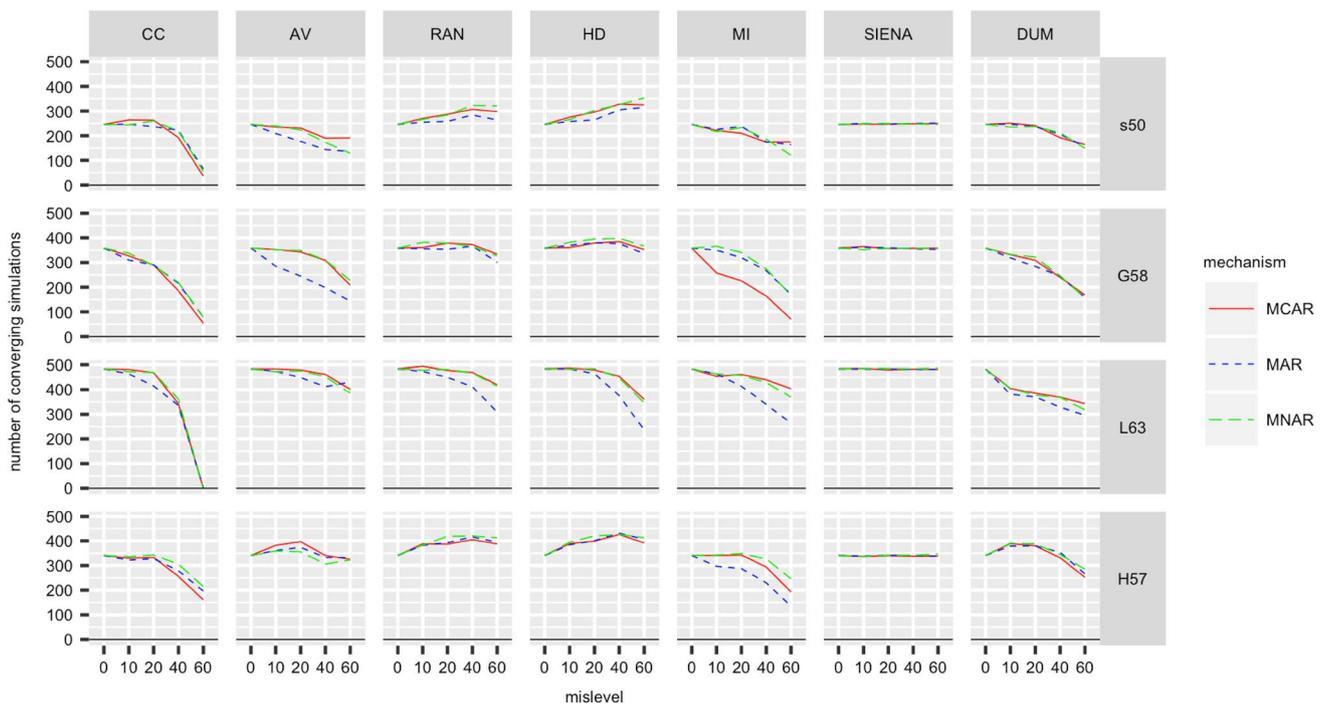


Fig. 2 Number of converging simulation runs

complete case analysis. Although MI imputes observed information (based on donor cases), the variation in imputed values is probably too large to lead to stable estimation. The two single imputation methods based on donor cases (RAN and HD) often result in increased numbers of convergent runs, indicating that they insert information that benefits model estimation, which may not actually reflect true data processes.

The default SIENA method shows rather strong convergence performance, even for high levels of missingness. However, using SIENA with the dummy option (DUM) leads to convergence problems with higher levels of missingness. The explanation of this difference might be due to the manner SIENA treats missing data. In the case of the default, the mode of the variable is imputed for the missing behavior data, and this value is allowed to change in the Markov sequence that models the behavior between the waves. This means that the incomplete cases still participate in the coevolution of behavior and network between the waves, though they are excluded in the procedure for parameter estimation. In the dummy method, any change of the missing behavior is effectively prohibited, limiting the coevolution process between waves. Especially for higher fractions of missing data this may lead to convergence problems.

The overall conclusion is that the SIENA method is least affected by convergence problems. Single imputation gives acceptable convergence rates for small numbers of missing data, but for higher percentages the performance gets worse.

The same pattern can be seen for the multiple imputation method, however the effects are even larger, especially for higher levels of missingness.

5.2 Parameter bias

The average relative biases of the estimated model parameters are presented in Fig. 3a–d, which consists of one subfigure for each data set. Each of these subfigures in turn contains a number of graphs for different combinations of method and estimated parameter. These figures show the average relative bias of the estimated parameters as a function of missingness level for the three different missingness mechanisms.

From Fig. 3a–d it can be seen that the missingness mechanisms (MCAR, MAR, and MNAR) seem to have little impact on the relative parameter bias. With some minor exceptions, there is some variation in performance, single imputation methods (AV, RAN, and HD) perform rather similarly. Depending on the type of the relation and the strength of the effect in the data, some effects will be more sensitive to missing data than others, leading to a variation in relative biases. The performance of the multiple imputation procedure is comparable to single imputations in a number of cases, but sometimes outperforms them, for example in the L63 data set. The default SIENA method clearly outperforms the other methods. The figures also indicate that the performance of this procedure seems to

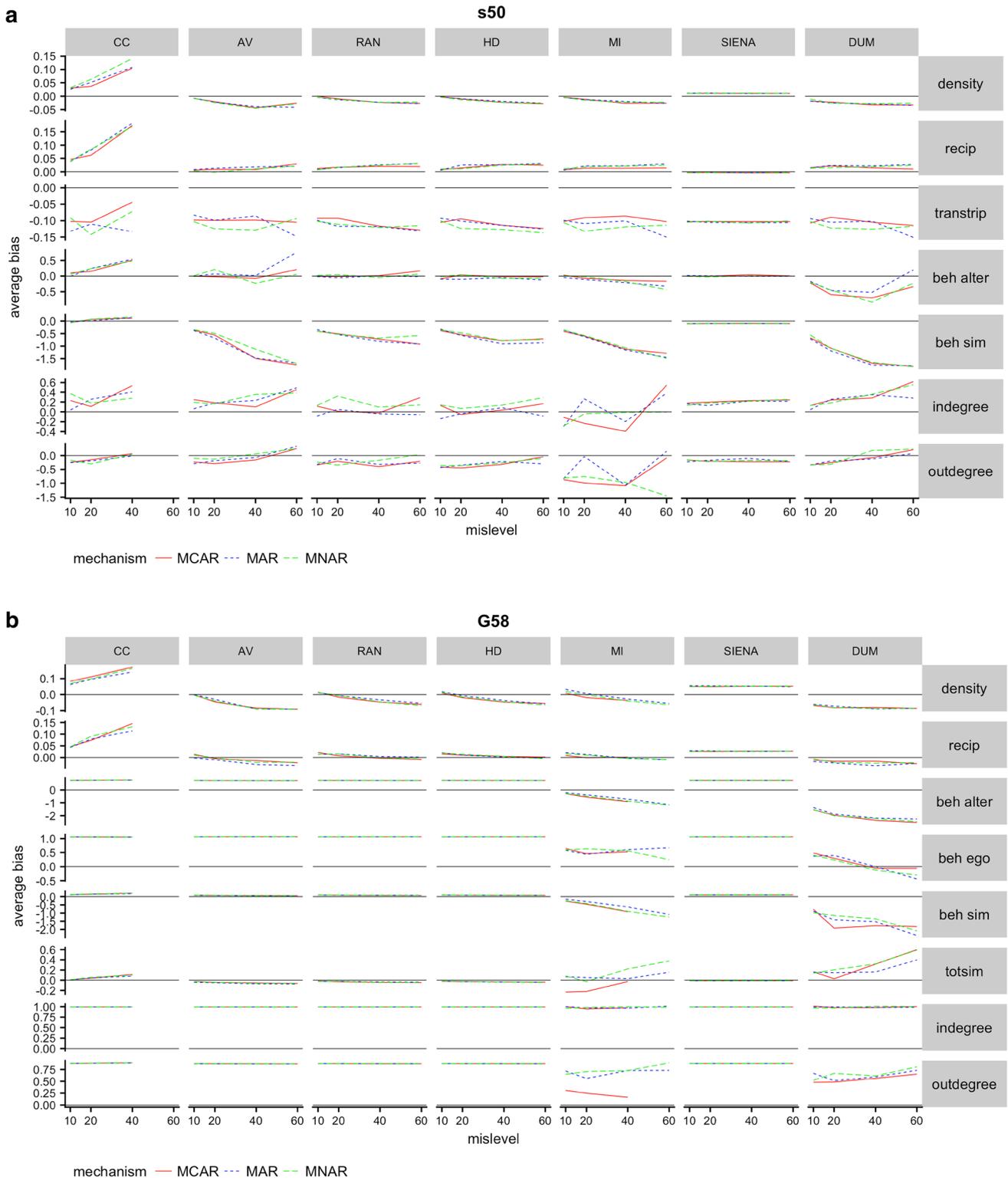


Fig. 3 **a** Data set s50. Relative average bias in all data sets, for all missing data methods (columns) and parameters of the corresponding base models (rows), mechanisms (lines in figures), and percentages of missing data (*x*-axis). **b** Data set G58. Relative average bias in all data sets, for all missing data methods (columns) and parameters of the corresponding base models (rows), mechanisms (lines in figures), and percentages of missing data (*x*-axis). **c** Data set L63. Relative

average bias in all data sets, for all missing data methods (columns) and parameters of the corresponding base models (rows), mechanisms (lines in figures), and percentages of missing data (*x*-axis). **d** Data set H57. Relative average bias in all data sets, for all missing data methods (columns) and parameters of the corresponding base models (rows), mechanisms (lines in figures), and percentages of missing data (*x*-axis)

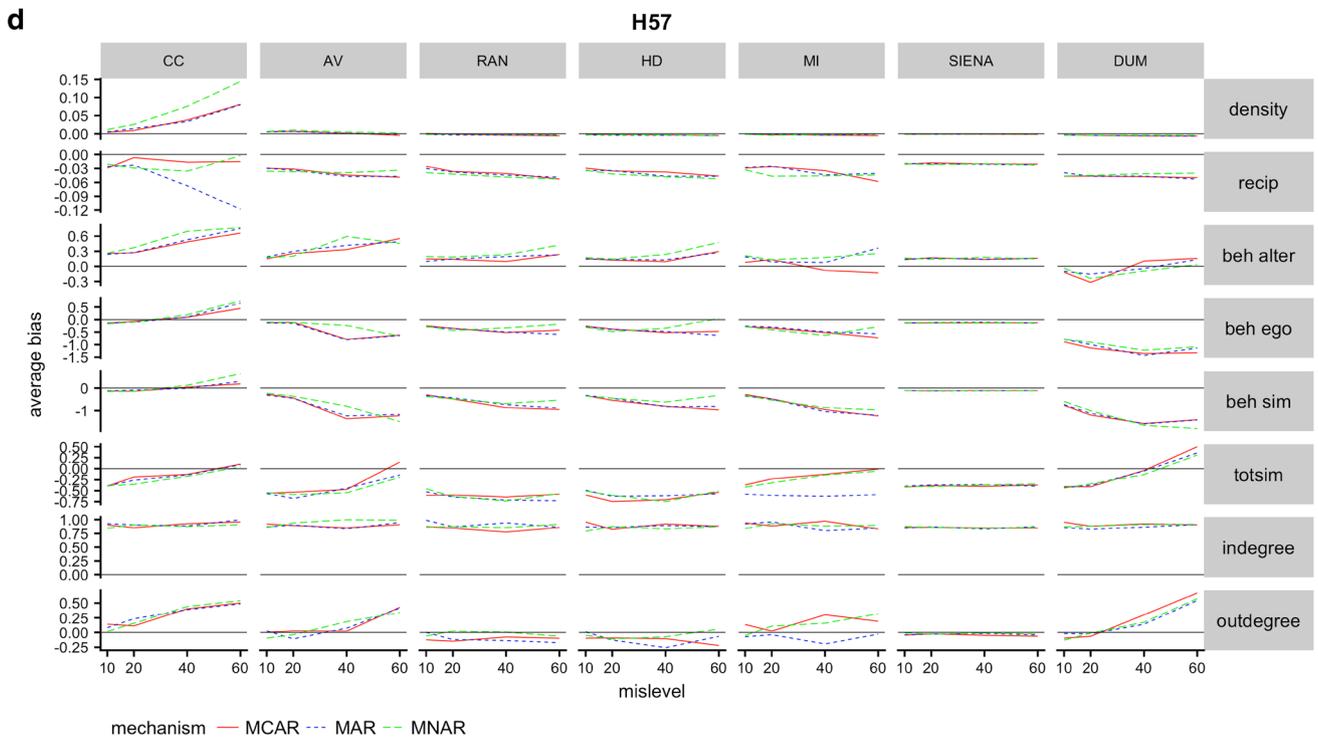
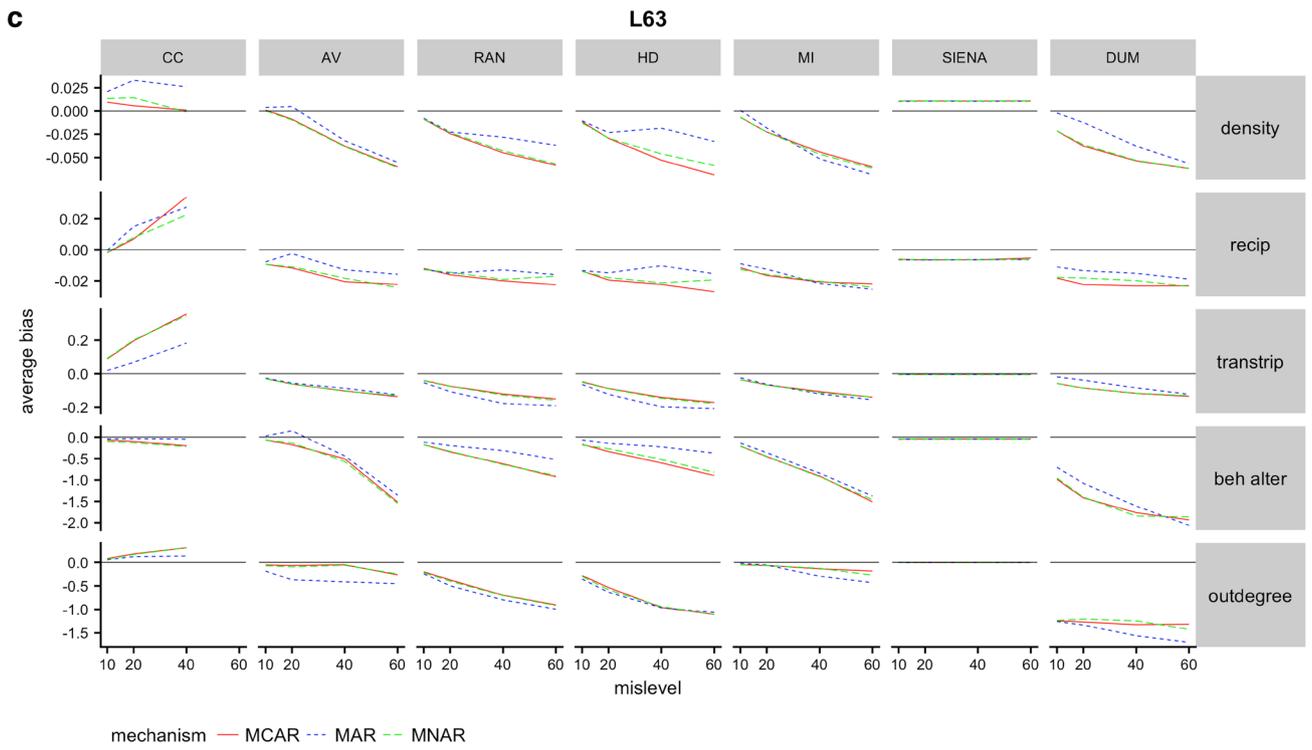


Fig. 3 (continued)

Table 3 Partial eta squared values from factorial ANOVAs for each combination of data set and model parameter to compare the main effects of missingness mechanism (MCAR, MAR, MNAR), level of missingness (10, 20, 40, 60%), and treatment method (CC, AV, RAN, HD, MI, SIENA, DUM), and their two-way and three-way interactions on the average relative bias

	Density	Recip	Transtrip	Beh alter	Beh ego	Beh sim	Tot sim	Indegree	Outdegree
s50									
Mechanism	0.00	0.00	0.00	0.00	–	0.00	–	0.00	0.00
Mislevel	0.01	0.01	0.00	0.00	–	0.04	–	0.00	0.00
Method	0.23	0.09	0.00	0.04	–	0.25	–	0.03	0.06
Mechanism*mislevel	0.00	0.00	0.00	0.00	–	0.00	–	0.00	0.00
Mechanism*method	0.00	0.00	0.00	0.00	–	0.00	–	0.00	0.00
Mislevel*method	0.13	0.04	0.00	0.01	–	0.09	–	0.00	0.00
Mechanism*mislevel*method	0.00	0.00	0.00	0.00	–	0.00	–	0.00	0.00
G58									
Mechanism	0.00	0.00	–	0.00	0.00	0.00	0.00	0.00	0.00
Mislevel	0.00	0.00	–	0.01	0.00	0.00	0.01	0.00	0.00
Method	0.28	0.11	–	0.86	0.16	0.32	0.23	0.00	0.04
Mechanism*mislevel	0.00	0.00	–	0.00	0.00	0.00	0.00	0.00	0.00
Mechanism*method	0.00	0.00	–	0.01	0.00	0.00	0.01	0.00	0.02
Mislevel*method	0.08	0.04	–	0.14	0.01	0.07	0.03	0.00	0.01
Mechanism*mislevel*method	0.00	0.00	–	0.00	0.00	0.00	0.00	0.00	0.00
L63									
Mechanism	0.02	0.00	0.01	0.02	–	–	–	–	0.00
Mislevel	0.01	0.01	0.04	0.08	–	–	–	–	0.01
Method	0.02	0.03	0.26	0.29	–	–	–	–	0.27
Mechanism*mislevel	0.01	0.00	0.00	0.00	–	–	–	–	0.00
Mechanism*method	0.01	0.00	0.02	0.01	–	–	–	–	0.00
Mislevel*method	0.06	0.04	0.06	0.10	–	–	–	–	0.08
Mechanism*mislevel*method	0.01	0.01	0.01	0.01	–	–	–	–	0.00
H57									
Mechanism	0.00	0.00	–	0.00	0.00	0.00	0.00	0.00	0.00
Mislevel	0.01	0.00	–	0.01	0.00	0.03	0.01	0.00	0.01
Method	0.07	0.00	–	0.02	0.13	0.18	0.03	0.00	0.02
Mechanism*mislevel	0.00	0.00	–	0.00	0.00	0.00	0.00	0.00	0.00
Mechanism*method	0.01	0.00	–	0.00	0.00	0.00	0.00	0.00	0.00
Mislevel*method	0.05	0.00	–	0.00	0.03	0.07	0.01	0.00	0.01
Mechanism*mislevel*method	0.00	0.00	–	0.00	0.00	0.00	0.00	0.00	0.00

To achieve convergence in the model estimation, different parameters have been deleted from the model specifications, resulting in not all effects being present in each factorial ANOVA

be rather independent of the level of missingness, whereas most methods show deteriorating performance for higher level of missingness. This is likely to be caused by the way SIENA deals with missing behavior values: while a value is imputed for the missing observation and behavior may change between the waves, the parameter estimation is based upon the non-missing cases only. This gives the SAOM a high level of flexibility and the possibility to adapt, while at the same moment only taking into account observed values for parameter estimation.

Factorial ANOVAs were performed to find the most important factors that affect the average relative bias of the estimated parameters. To reduce the amount of effects to

be estimated and to increase clarity of interpretation, the ANOVAs were performed for each combination of data set and model parameter separately. That is, the main effects on bias of missingness mechanism, level of missingness, and treatment method were estimated, as well as all two-way and three-way interactions. The partial eta-squared values, η_p^2 , of main, two-way and three-way effects are presented per parameter and per data set in Table 3.

In addition, for each data set a linear regression was performed to predict the relative biases of the estimated parameters using dummy variables representing the different categorical factors (only main effects, no interactions). In this analysis, the default SIENA method was chosen as

Table 4 Estimated parameters (with standard errors) of the regressions predicting the average relative bias using dummy variables for missing data method, missingness mechanism, missingness level, and model parameter, for all four data sets

	s50	G58	L63	H57
Intercept	0.063 (0.006)	0.075 (0.003)	0.190 (0.004)	0.067 (0.006)
CC	0.140 (0.005)	0.062 (0.003)	0.080 (0.004)	0.139 (0.005)
AV	-0.059 (0.005)	-0.037 (0.003)	-0.120 (0.004)	-0.063 (0.005)
RAN	-0.094 (0.005)	-0.022 (0.003)	-0.200 (0.004)	-0.130 (0.006)
HD	-0.109 (0.005)	-0.022 (0.003)	-0.214 (0.004)	-0.129 (0.006)
MI	-0.287 (0.007)	-0.325 (0.003)	-0.176 (0.004)	-0.174 (0.006)
DUM	-0.139 (0.005)	-0.237 (0.003)	-0.381 (0.004)	-0.185 (0.006)
MAR	-0.001 (0.004)	0.004 (0.002)	0.025 (0.003)	-0.002 (0.004)
MNAR	0.002 (0.004)	0.002 (0.002)	0.001 (0.003)	0.014 (0.004)
mis20	-0.020 (0.004)	-0.001 (0.002)	-0.036 (0.003)	-0.024 (0.004)
mis40	-0.019 (0.004)	0.013 (0.002)	-0.097 (0.003)	-0.020 (0.004)
mis60	0.043 (0.004)	0.034 (0.002)	-0.127 (0.003)	0.066 (0.004)
Recip	0.033 (0.005)	0.004 (0.003)	0.019 (0.003)	-0.043 (0.006)
Transtrip	-0.111 (0.005)	-	-0.044 (0.003)	-
Beh_alter	0.002 (0.006)	0.468 (0.003)	-0.395 (0.003)	0.206 (0.006)
Beh_ego	-	0.912 (0.003)	-	-0.357 (0.006)
Beh_sim	-0.466 (0.006)	-0.014 (0.003)	-	-0.445 (0.006)
Totsim	-	0.059 (0.003)	-	0.031 (0.006)
Indegree	0.596 (0.006)	0.969 (0.003)	-	0.910 (0.006)
Outdegree	0.449 (0.006)	0.818 (0.003)	-0.269 (0.003)	0.340 (0.006)

the reference for method, because Fig. 3a–d indicates this to be the best performing method. For the missingness mechanism, we chose MCAR, as a theoretical ideal situation. For the level of missingness, we chose 10%, the lowest percentage. And for parameter, at random, density. The results of the regressions are presented in Table 4.

The first observation from the factorial ANOVA is that the η_p^2 values for the missingness mechanism are all very small. This indicates that of the variances in bias, the proportions associated with the missingness mechanism are very small. The largest values are found in the L63 data set with $\eta_p^2=0.015$ for the density effect and $\eta_p^2=0.017$ for the behavior alter effect. This is confirmed in the regression on the dummy variables in Table 4, where MAR and MNAR only add minor and often insignificant differences to MCAR. The results in Fig. 3a–d also show often little differences between the missingness methods. The conclusion is that the differences between missingness mechanisms are very small for almost all cases and that the missingness mechanism is not a major factor to consider for the selection of a method to deal with missing data.

A second conclusion to be drawn from the ANOVAs is that the level of missingness is only limited associated with the variation in parameter bias. Large partial eta-squared values are $\eta_p^2=0.044$ for transitive triplets (transtrip) and $\eta_p^2=0.080$ for behavior alter effects in the L63 data set, $\eta_p^2=0.037$ for the behavior similarity effect in the s50 data set, and $\eta_p^2=0.026$ for the behavior similarity effects in the H57 data set. These results can also be seen in Fig. 3a–d,

where for the mentioned cases an increase in (negative) average relative bias can be seen.

Much larger partial eta-squared values can be found for method. Primarily for the main effect of method (e.g., $\eta_p^2=0.231$ and 0.283 for density in the s50 and the G58 data set, resp.), but as well for some two-way interactions between method and missingness level (e.g., for the behavior alter effect, $\eta_p^2=0.138$ in the G58 data set and $\eta_p^2=0.103$ in the L63 data set, and for the behavior similarity effect, $\eta_p^2=0.088$, 0.070 , and 0.066 for the s50, G58, and H57 data sets, resp.). That the method to deal with missing data is the largest source of variation in parameter bias indicates that choosing the right method is the most important concern.

Looking at the different methods, CC shows relatively large deviations from the true model parameters. Furthermore, in Fig. 3a–d it can be seen that the four imputation methods (AV, RAN, HD, and MI) show comparable patterns, and especially RAN and HD almost give similar results. In data set s50, MI shows similar results as RAN and HD. In data set L63, MI performs better for reciprocity, transitive triplets and outdegree. In G58, MI differs slightly from the other three methods, showing less deviation for behavior ego, but more deviation for behavior similarity, total similarity, and outdegree. This is confirmed by the regression analyses, where again we see that RAN and HD show almost identical effects.

In most cases SIENA shows the smallest bias, and DUM is similar to the imputation methods. This is not unexpected, as with dummies the value of missing behavior is fixed to

a constant level using a high penalty for change. SIENA imputes the mode and carries out a simulation as if the data set was complete. The actual parameter estimation is then based on the non-missing data. The difference with the dummy method is the allowed change in the underlying sequences of mini steps. It appears that denying these micro changes for missing data, which effectively means putting a lock on behavior, results in larger biases relative to the true parameter.

5.3 Coverage

To evaluate the distribution of the estimated parameters, the proportion of population parameters (base model) within two-standard-errors distance from the estimated parameter was calculated. In case of a normal distribution of estimated parameters, this distance would be smaller than two standard errors in approximately 95% of the cases. Although the distribution of estimated parameters in Siena is unknown, we will use this procedure to approximate parameter coverage. Similar to the relative biases, this is based on combinations with at least 100 convergent runs only.

The coverage results are presented in Fig. 4a–d. These figures show the proportions of population parameters that are within two-standard-errors distance from the corresponding estimated parameters. In each plot, the horizontal line at 0.95 gives the 95% normal-distribution benchmark.

The first observation is that there is hardly a difference between the three missingness mechanisms. Inspection of the missing data methods shows that in most cases CC leads to 95% of the population parameters within the expected boundary, although it should be noted that for high percentages missing (60%), the proportions could not be calculated in three of the four data sets (due to low numbers of convergent model runs). Only the L63 transitive triplets parameter, and the G58 behavior alter and ego effects, indegree, and outdegree parameters show an unexpected low proportion of smaller than 2 SE deviations. For the G58 data, these parameters show similar poor results for all other methods as well.

In general, the imputation methods perform similar or worse than CC, especially for higher levels of missing data. The single imputations AV, RAN, and HD give similar results (again RAN and HD are almost identical in their patterns) and are not able to give acceptable parameter coverage for high levels of missing data. Multiple imputation can only do a little better, indicating that the number of imputations ($m=5$) is probably too low. In the L63 data, MI clearly outperforms the single imputation for parameter coverage of the transitive triples and outdegree effects and is slightly better for other effects. MI also clearly outperforms CC, AV, RAN, and HD in the G58 and H57 data sets.

The outcomes of SIENA are always at the 0.95 level, except for a number of effects in the G58 data set. The

dummy method is on a number of individual cases on the 0.95 level, but often clearly much lower (for some parameters in all data sets, e.g., s50 behavior alter and behavior sim, or L63 all parameters but reciprocity). This is not surprising as the dummy effect prevents development of the behavior between waves, and so hampers effects related to behavior.

There is no clear indication that one parameter is experiencing more coverage problems than others. All parameters show poor results in some conditions. For example, indegree and outdegree show satisfactorily results in the s50 and H57 data sets, poor results for most methods in the G58 data set, and mixed results in the L63 data set. Overall, the conclusion is that SIENA outperforms other methods.

There are two notable effects of type of data set. First, the L63 data shows more variation in coverage than the other data sets, with low coverage levels for high missingness levels. Only for the SIENA method coverage is stable and at the desired level. Second, the G58 data shows extremely low coverage for some behavior effects (alter, ego, indegree, outdegree), for all methods. These effects were already non-significant in the base model (in the sense that the estimated value is smaller than two times the standard error), and the simulations show wildly varying results. Multiple imputation can partly compensate for this by taking into account the between-imputations variance.

6 Discussion

Missing data is a challenge for network researchers. While much research has focused on the effect of missing actors, we addressed the influence of missing actor behavior in longitudinal network studies. In particular we focused on strategies to deal with missing behavior data in SAOMs. These strategies are based on general principles to deal with missing data and can be categorized as complete cases only, imputation-based, and model-based methods. We have used three criteria to evaluate these strategies: estimation convergence, relative parameter bias, and coverage. We argue that relative parameter bias is the most important of these three criteria, as it reflects the accuracy of the parameter estimates. Coverage describes the proportion of two-standard-errors intervals around the estimated parameter that contain the original parameter (true parameter of the base model which generated the simulated data). We have to realize that the distribution of the parameter estimates is unknown, which implies that a priori we cannot expect 95% of the estimated intervals to contain the true parameter. Despite this caveat, the proportion found is an indication of the spread and stability of outcomes. Therefore, we consider coverage the second criteria to apply. Thirdly, convergence is an indication of correct model specification. Poor convergence means that the data set contains not enough information to estimate the

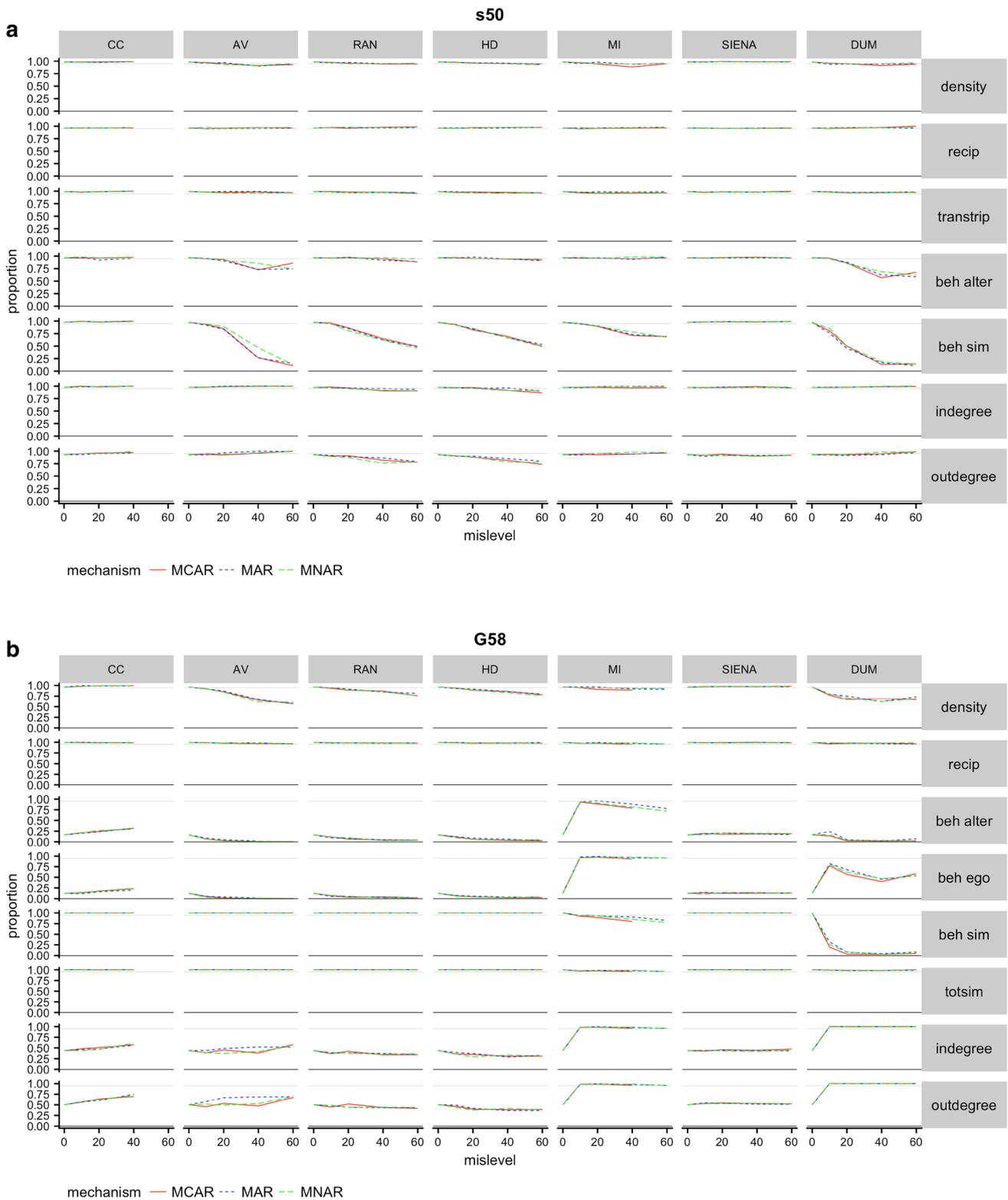


Fig. 4 a Data set s50. Proportion of runs with difference between population parameter and estimated parameter smaller than two standard errors. **b** Data set G58. Proportion of runs with difference between population parameter and estimated parameter smaller than two standard errors. **c** Data set L63. Proportion of runs with differ-

ence between population parameter and estimated parameter smaller than two standard errors. **d** Data set H57. Proportion of runs with difference between population parameter and estimated parameter smaller than two standard errors

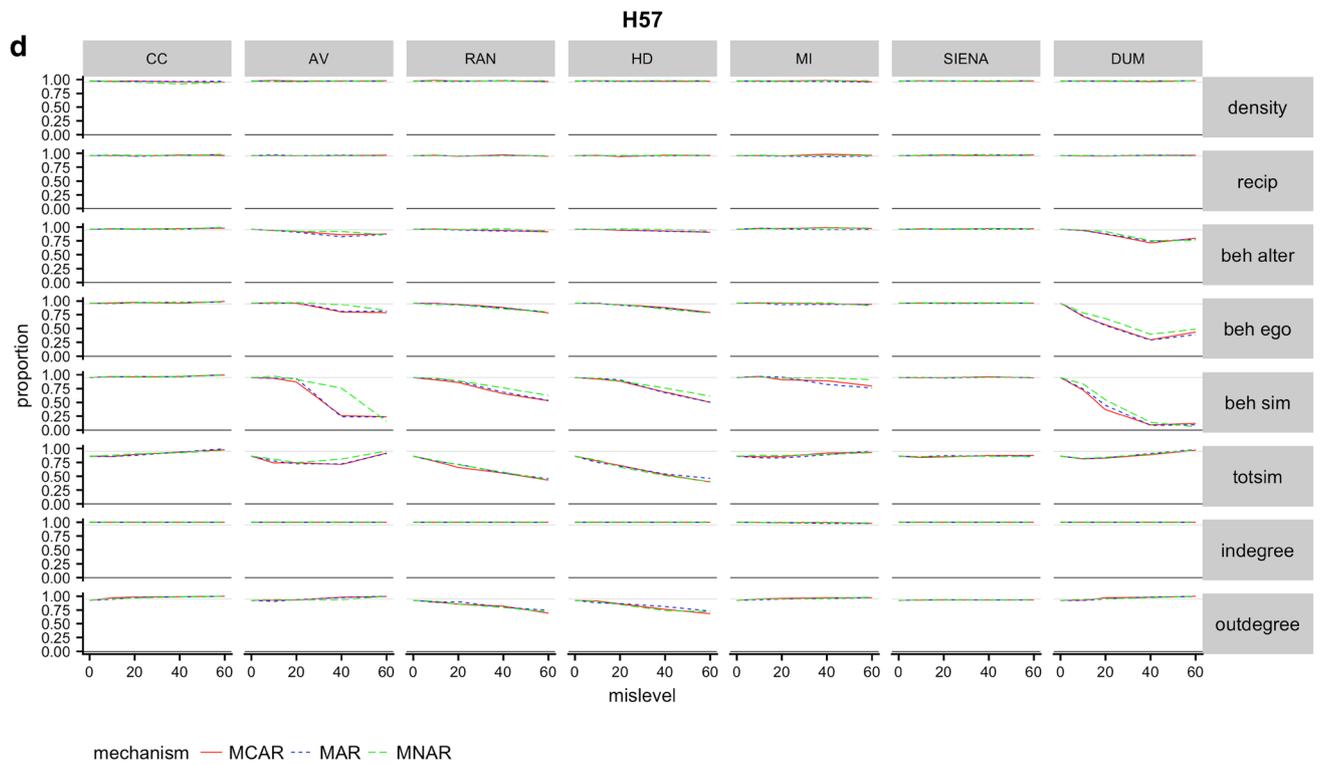
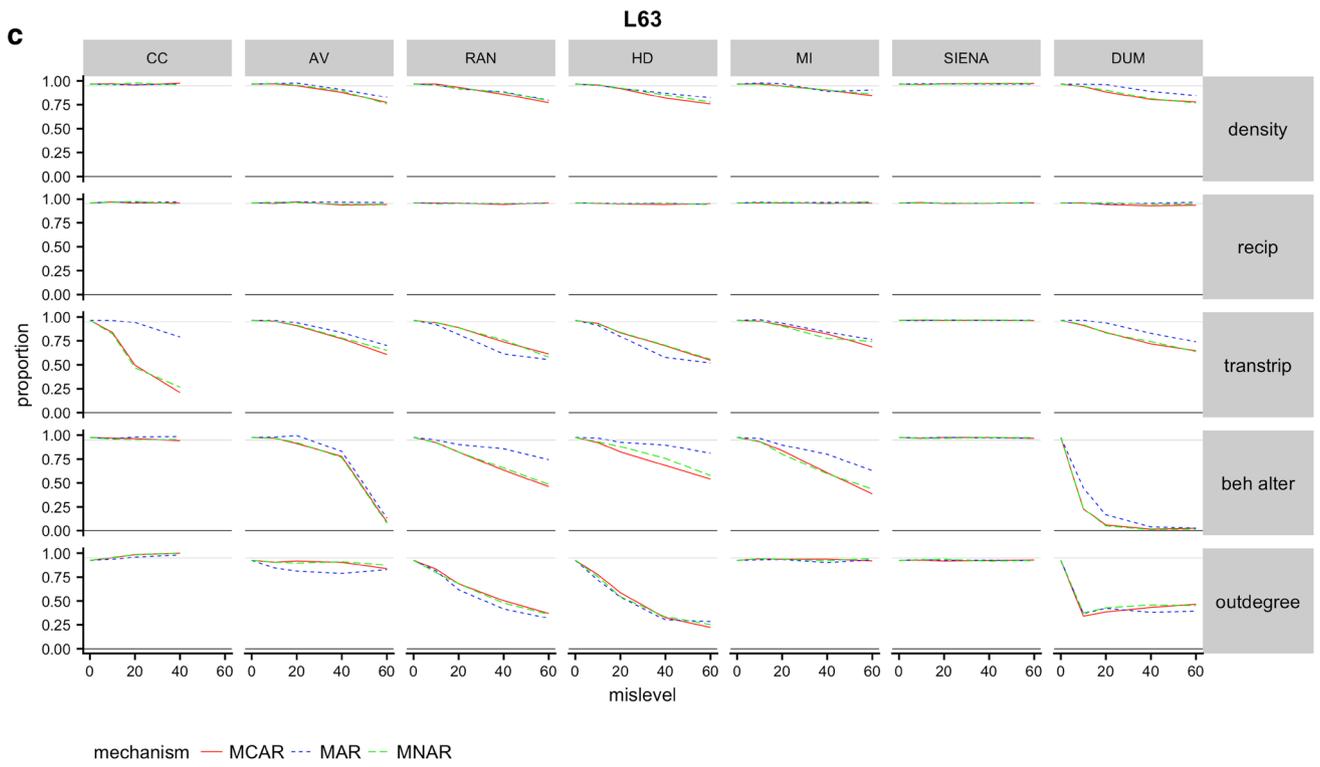


Fig. 4 (continued)

model parameters properly. This might be an indication the chosen strategy leads to a too strong loss of information as we have witnessed for the complete cases strategy in the simulation, or to an imputation that represents the missing information incompletely or imperfectly.

Regarding the relative parameter bias, the results show that the methods based on single imputation are roughly comparable. Multiple imputation sometimes outperforms the single imputations but performs worse in other cases. In most cases, the default SIENA procedure shows the smallest bias and the SIENA dummy method shows strongest parameter bias. These latter results show that the dummy method is too restrictive in allowing change between consecutive waves, as missing actors are not allowed to make changes, thereby biasing parameter estimates. The default SIENA procedure is more flexible and missing actors do influence the coevolution process, which leads to better results. The default SIENA procedure is more a model-based approach to deal with missing data, in that it estimates the missing values in the course of parameter estimation (i.e., in simulating the coevolution process) and missing actors do have an indirect influence on parameter estimates. However, contrary to the dummy approach or the imputation-based strategies, the final estimation is based only on the observed cases. This is shown to lead to better results (i.e., smaller biases in parameter estimates).

Single imputations often show a poor parameter coverage, especially for high levels of missingness and for parameters related to (the missing) behavior. Multiple imputation often, but not always, outperforms single imputation. This also shows that the number of imputations used in the simulations ($m = 5$) is not large enough. The SIENA dummy method often gives coverage results below the 0.95 level, and gives in general similar results as imputation, and in some cases even worse. For the default SIENA method, however, in almost all combinations of data set and effect parameter, 95% of the two-standard-errors intervals around the estimated parameters contain the original parameter, making it the best performing procedure with respect to coverage.

The single donor-based imputation methods (RAN and HD) often see an increase of convergent runs for higher levels of missingness. This suggests information is inserted that benefits model estimation, that is, there are enough changes between consecutive waves to result in some converged solution. This solution, however, often is biased and coverage is poor. Mean imputation and especially multiple imputation give poorer results with respect to convergence. The latter result may be due to large variation between imputations, indicating that there are large differences between imputation runs, which do not allow for stable estimation of SAOMs. The SIENA methods perform better, where SIENA with the dummy option leads to convergence problems with higher levels of missingness, but the default SIENA method

shows rather strong convergence performance, even for high levels of missingness.

We observed only a limited influence of the missingness mechanisms MCAR, MAR (probability of missingness proportional to outdegree), and MNAR (probability of missingness proportional to behavior). This suggests that the performance of each treatment strategy is mostly unaffected by the used missingness mechanisms. There was also a small effect of the data set used, especially on the coverage criterion, but there were no methods that performed substantially better in one data set than another.

Taking all three criteria into consideration, we recommend the default SIENA procedure as the optimal strategy currently available to deal with missing behavior data. First, it leads to the smallest average parameter bias. Secondly, for almost all combinations of effect parameter and data set we investigated, 95% of the two-standard-errors intervals around the estimated parameters contain the original parameter. And thirdly, it has the best convergence performance, which besides being an indication of operational strength, indicates that other methods are less capable of dealing with the loss of information due to the missing behavior data.

The study has a number of limitations. First, is the use of real-life data sets instead of simulated data. As it is impossible to draw clear conclusions about the effect of data characteristics on the performance of the missing data procedures, the potential influence of the characteristics has to be explored further. For example, closure effects are more likely to be affected in low-density networks than in high-density networks, because the impact of missing data will be felt more severely. This holds in particular for friendship networks. Although the explored data sets differed in terms of network characteristics like density, type of ties (friendship/advice) and behavior variables, we have not been able to establish a relation between these characteristics and their impact on the three evaluation criteria we applied.

Secondly, one likely reason for the worse performance of the imputation methods, might be their neglect of the influence of network structure. To deal with missing ties in cross-sectional network studies, Krause et al. (2018b) proposed a multiple imputation based on Bayesian ERGMS, which uses the information contained in the network structure. Applying this approach to missing behavior data might lead to improved performance, because better use is made of the information contained in the data set.

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